Chapter 1: Units and Measurements

EXERCISES [PAGES 14 - 15]

Exercises | Q 1. (i) | Page 14

Choose the correct option.
$[L^1M^1T^{-1}]$ is the dimensional formula for

1. Velocity
2. Acceleration
3. **Force**
4. Work

SOLUTION

$[L^1M^1T^{-1}]$ is the dimensional formula for **force.**

Exercises | Q 1. (ii) | Page 14

Choose the correct option.
The error in the measurement of the sides of a rectangle is 1%. The error in the measurement of its area is

1%

$\frac{1}{2}\%$

2%

None of the above

SOLUTION

The error in the measurement of the sides of a rectangle is 1%. The error in the measurement of its area is **2%**.

Explanation:

A = l × b

$$\therefore \frac{\triangle A}{A} = \frac{\triangle l}{l} + \frac{\triangle b}{b} = 1\% + 1\% = 2\%$$

Exercises | Q 1. (iii) | Page 14

Choose the correct option.

A light year is a unit of ______.

1. Time
2. Mass
3. **Distance**
4. Luminosity

SOLUTION

A light year is a unit of **distance.**

Exercises | Q 1. (iv) | Page 14

Choose the correct option.
Dimensions of kinetic energy are the same as that of ______.

1. Force
2. Acceleration
3. **Work**
4. Pressure

SOLUTION

Dimensions of kinetic energy are the same as that of **work**.

Exercises | Q 1. (v) | Page 14

Choose the correct option.
Which of the following is not a fundamental unit?

1. cm
2. kg
3. centigrade
4. **volt**

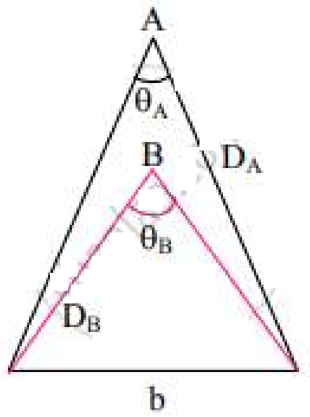
SOLUTION

volt

Exercises | Q 2. (i) | Page 14

Answer the following question.
Star A is farther than star B. Which star will have a large parallax angle?

SOLUTION



1. 'b' is constant for the two stars

   $\therefore \theta = \dfrac{1}{D}$

2. As star A is farther i.e., $D_A > D_B$

   $\Rightarrow \theta_A < \theta_B$

   Hence, star B will have a larger parallax angle than star A.

Exercises | Q 2. (ii) | Page 14

Answer the following question.

What are the dimensions of the quantity $l\sqrt{l/g}$, l being the length, and g the acceleration due to gravity?

SOLUTION

Quantity = $l \times \sqrt{\frac{l}{g}}$ ....(i)

gravitational acceleration, g = $\frac{\text{velocity}}{\text{time}}$

$\therefore$ g = $\frac{\text{distance}}{\text{time} \times \text{time}}$

Substituting in equation (i),

Quantity = $l \times \sqrt{\frac{l \times \text{time}^2}{\text{distance}}}$

$\therefore$ Dimensional formula of quantity

$$= [L] \times \frac{\left[L^{1/2}\right]\left[T^{2\times 1/2}\right]}{L^1/2} = [L] \times \left[T^1\right] = \left[L^1 T^1\right]$$

Exercises | Q 2. (iii) | Page 14

Define absolute error.

SOLUTION

1. For a given set of measurements of a quantity, the magnitude of the difference between mean value (Most probable value) and each individual value is called absolute error (Δa) in the measurement of that quantity.

2. absolute error = |mean value - measured value|

$\Delta a_1 = |a_{mean} - a_1|$

Similarly, $\Delta a_2 = |a_{mean} - a_2|$, ...$\Delta a_n = |a_{mean} - a_n|$

Exercises | Q 2. (iii) | Page 14

Define Mean absolute error.

SOLUTION

For a given set of measurements of the same quantity, the arithmetic mean of all the absolute errors is called mean absolute error in the measurement of that physical quantity.

$$\triangle \mathrm{a}_{\mathrm{mean}} = \frac{\triangle \mathrm{a}_1 + \triangle \mathrm{a}_2 + \ldots\ldots + \triangle \mathrm{a}_\mathrm{n}}{\mathrm{n}} = \frac{1}{\mathrm{n}} \sum_{i=1}^{n} \triangle a_i$$

Exercises | Q 2. (iii) | Page 14

Define mean percentage error.

SOLUTION

The relative error represented by percentage (i.e., multiplied by 100) is called the percentage error.

$$\text{Percentage error} = \frac{\triangle \mathrm{a}_{\mathrm{mean}}}{\mathrm{a}_{\mathrm{mean}}} \times 100\%$$

Exercises | Q 2. (iv) | Page 14

Answer the following question.
Describe what is meant by significant figures and order of magnitude.

SOLUTION

1. Significant figures in the measured value of a physical quantity is the sum of reliable digits and the first uncertain digit.
   OR
   The number of digits in a measurement about which we are certain, plus one additional digit, the first one about which we are not certain is known as significant figures or significant digits.

2. The larger the number of significant figures obtained in a measurement, the greater is the accuracy of the measurement. The reverse is also true.

3. If one uses the instrument of smaller least count, the number of significant digits increases.

Rules for determining significant figures:

1. All the non-zero digits are significant, for example, if the volume of an object is 178.43 cm^3, there are five significant digits which are 1,7,8,4 and 3.

2. All the zeros between two nonzero digits are significant, eg., m = 165.02 g has 5 significant digits.

3. If the number is less than 1, the zero/zeroes on the right of the decimal point and to the left of the first nonzero digit are not significant e.g. in 0.001405, the underlined zeroes are not significant. Thus the above number has four significant digits.

4. The zeroes on the right-hand side of the last nonzero number are significant (but for this, the number must be written with a decimal point), e.g. 1.500 or 0.01500 both have 4 significant figures each.
   On the contrary, if a measurement yields length L given as L = 125 m = 12500 cm = 125000 mm, it has only three significant digits.

Exercises | Q 2. (v) | Page 14

Answer the following question.
If the measured values of the two quantities are A ± ΔA and B ± ΔB, ΔA and ΔB being the mean absolute errors. What is the maximum possible error in A ± B?

SOLUTION

The maximum possible error in (A ± B) is (ΔA + ΔB).

Exercises | Q 2. (v) | Page 14

Answer the following question.

Show that if Z = $\frac{A}{B}$, $\frac{\triangle Z}{Z} = \frac{\triangle A}{A} + \frac{\triangle B}{B}$

SOLUTION

Errors in divisions:

Suppose, Z = $\frac{A}{B}$ and measured values of A and B are (A ± ΔA) and (B ± ΔB) then,

$$Z \pm \Delta Z = \frac{A \pm \triangle A}{B \pm \triangle B}$$

$$\therefore Z\left(1 \pm \frac{\triangle Z}{Z}\right) = \frac{A[1 \pm (\Delta A/A)]}{B[1 \pm (\Delta B/B)]}$$

$$= \frac{A}{B} \times \frac{1 \pm (\Delta A/A)}{1 \pm (\Delta B/B)}$$

As, $\frac{\triangle B}{B} \ll 1$, expanding using Binomial theorem,

$$Z\left(1 \pm \frac{\triangle Z}{Z}\right) = Z\left(1 \pm \frac{\triangle A}{A}\right) \times \left(1 \mp \frac{\Delta B}{B}\right) \quad .... \left(\because \frac{A}{B} = Z\right)$$

$$\therefore 1 \pm \frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \times \frac{\Delta B}{B}$$

Ignoring term $\frac{\Delta A}{A} \times \frac{\Delta B}{B}$, $\frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B}$

This gives four possible values of $\frac{\Delta Z}{Z}$ as

$$\left(+\frac{\Delta A}{A} - \frac{\Delta B}{B}\right), \left(+\frac{\Delta A}{A} + \frac{\Delta B}{B}\right), \left(-\frac{\Delta A}{A} - \frac{\Delta B}{B}\right) \text{ and } \left(-\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

$\therefore$ Maximum relative error of $\frac{\Delta Z}{Z} \pm \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Thus, when two quantities are divided, the maximum relative error in the result is the sum of relative errors in each quantity.

Exercises | Q 2. (vi) | Page 14

Answer the following question.

Derive the formula of the kinetic energy of a particle having mass ‘m’ and velocity ‘v’, using dimensional analysis.

SOLUTION

The kinetic energy of a body depends upon mass (m) and velocity (v) of the body.

Let K.E. $\propto m^x v^y$

$\therefore$ K.E. = $km^x v^y$ .....(1)

where, k = dimensionless constant of proportionality. Taking dimensions on both sides of equation (1),

$$\left[L^2M^1T^{-2}\right] = \left[L^0M^1T^0\right]^x \left[L^1M^0T^{-1}\right]^y$$

$$= \left[L^0M^xT^0\right]\left[L^yM^0T^{-y}\right]$$

$$= \left[L^{0+y}M^{x+0}T^{0-y}\right]$$

$$\left[L^2M^1T^{-2}\right] = \left[L^yM^xT^{-y}\right] \quad ....(2)$$

Equating dimensions of L, M, T on both sides of equation (2),

x = 1 and y = 2

Substituting x, y in equation (1), we have

K.E. = kmv^2

Exercises | Q 3. (i) | Page 14

Solve the numerical example.
The masses of two bodies are measured to be 15.7 ± 0.2 kg and 27.3 ± 0.3 kg. What is the total mass of the two and the error in it?

SOLUTION

Given: A ± ΔA = 15.7 ± 0.2 kg and
B ± ΔB = 27.3 ± 0.3 kg.

To find: Total mass (Z), and total error (ΔZ)

Formulae: i. Z = A + B
ii. ± ΔZ = ± ΔA ± ΔB

Calculation: From formula (i),

Z = 15.7 + 27.3 = 43 kg

From formula (ii),

± ΔZ = (± 0.2) + (± 0.3)

= ± (0.2 + 0.3)

= ± 0.5 kg

Total mass is **43 kg** and total error is **± 0.5 kg.**

Exercises | Q 3. (ii) | Page 14

Solve the numerical example.

The distance travelled by an object in time (100 ± 1) s is (5.2 ± 0.1) m. What are the speed and its maximum relative error?

SOLUTION

Given: Distance $(D \pm \Delta D) = (5.2 \pm 0.1)$ m,

time $(t \pm \Delta t) = (100 \pm 1)$ s.

To find: Speed (v), the maximum relative error $\left(\frac{\triangle v}{v}\right)$

Formulae: i. $v = \frac{D}{t}$

ii. $\frac{\triangle v}{v} = \pm\left(\frac{\triangle D}{D} + \frac{\triangle t}{t}\right)$

Calculation: From formula (i),

$v = \frac{5.2}{100} = 0.052$ m/s

From formula (ii),

$$\left.\frac{\triangle v}{v}\right) = \pm\left(\frac{0.1}{5.2} + \frac{1}{100}\right)$$

$$= \pm\left(\frac{1}{52} + \frac{1}{100}\right) = \pm\frac{19}{650}$$

$= \pm\ 0.029$ m/s

The speed is **0.052 m/s** and its maximum relative error is **± 0.029 m/s**.

Exercises | Q 3. (iii) | Page 14

Solve the numerical example.

An electron with charge e enters a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$. The velocity is perpendicular to the magnetic field. The force on the charge is given by $\left|\vec{F}\right|$ = B e v.

Obtain the dimensions of $\vec{B}$.

SOLUTION

Given: $\left|\overrightarrow{F}\right|$ = B e v

Considering only magnitude, given equation is simplified to,

F = B e v

$\therefore$ B = $\dfrac{\mathrm{F}}{\mathrm{e\ v}}$

but, F = ma = m $\times\ \dfrac{\text{distance}}{\text{time}^2}$

$\therefore\ [\mathrm{F}] = [\mathrm{M}^1] \times \left[\dfrac{\mathrm{L}^1}{\mathrm{T}^2}\right]$

= $[L^1M^1T^{-2}]$

Electric charge, e = current × time

$\therefore$ [e] = $[I^1T^1]$

Velocity v = $\dfrac{\text{distance}}{\text{time}}$

$\therefore\ [\mathrm{v}] = \left[\dfrac{\mathrm{L}}{\mathrm{T}}\right] = [\mathrm{L}^1\mathrm{T}^{-1}]$

Now, [B] = $\left[\dfrac{\mathrm{F}}{\mathrm{e\ v}}\right]$

$$= \frac{[\mathrm{L}^1\mathrm{M}^1\mathrm{T}^{-2}]}{[\mathrm{T}^1\mathrm{I}^1]\,[\mathrm{L}^1\mathrm{T}^{-1}]}$$

$\therefore$ [B] = $[L^0M^1T^{-2}I^{-1}]$

Exercises | Q 3. (iv) | Page 14

Solve the numerical example.

A large ball 2 m in radius is made up of a rope of square cross-section with edge length 4 mm. Neglecting the air gaps in the ball, what is the total length of the rope to the nearest order of magnitude?

SOLUTION

Volume of ball = Volume enclosed by rope.

$\frac{4}{3}\pi\ (\text{radius})^3$ = Area of cross-section of rope × length of rope.

$\therefore$ length of rope l = $\frac{\frac{4}{3}\pi r^3}{A}$

Given: r = 2 m and

Area = A = 4 × 4 = 16 mm^2 = 16 × 10^{-6} m^2

$$\therefore l = \frac{4 \times 3.142 \times 2^3}{3 \times 16 \times 10^{-6}}$$

$$= \frac{3.142 \times 2}{3} \times 10^6 \text{m}$$

$\approx 2 \times 10^6$ m.

$\therefore$ Total length of rope to the nearest order of magnitude = 10^6 m = 10^3 km

Exercises | Q 3. (v) | Page 14

Solve the numerical example.
Nuclear radius R has a dependence on the mass number (A) as R =1.3 × 10^{-16} A$^{1/3}$ m. For a nucleus of mass number A = 125, obtain the order of magnitude of R expressed in the meter.

SOLUTION

R = 1.3 × 10^{-16} × A$^{1/3}$ m

For A = 125

R = 1.3 × 10^{-16} × (125)$^{1/3}$

= 1.3 × 10^{-16} × (5^3)$^{1/3}$

= 1.3 × 10^{-16} × 5

$= 6.5 \times 10^{-16}$

$= 0.65 \times 10^{-15}$ m

∴ Order of magnitude = - 15

Exercises | Q 3. (vi) | Page 14

Solve the numerical example.
In a workshop, a worker measures the length of a steel plate with Vernier calipers having a least count 0.01 cm. Four such measurements of the length yielded the following values: 3.11 cm, 3.13 cm, 3.14 cm, 3.14 cm. Find the mean length, the mean absolute error, and the percentage error in the measured value of the length.

SOLUTION

Given: a_1 = 3.11 cm, a_2 = 3.13 cm,

a_3 = 3.14 cm, a_4 = 3.14 cm

Least count L.C. = 0.01 cm.

To find: i. Mean length (a_{mean})

ii. Mean absolute error $(\triangle a_{mean})$

iii. Percentage error.

Formulae: 1. $a_{mean} = \dfrac{a_1 + a_2 + a_3 + a_4}{4}$

2. $\triangle a_n = |a_{mean} - a_n|$

3. $\triangle a_{mean} = \dfrac{\triangle a_1 + \triangle a_2 + \triangle a_3 + \triangle a_4}{4}$

4. Percentage error = $\dfrac{\triangle a_{mean}}{a_{mean}} \times 100$

Calculation: From formula (i),

$a_{mean} = \dfrac{3.11 + 3.13 + 3.14 + 3.14}{4}$ = 3.13 cm

From formula (ii),

$\triangle a_1 = |3.13 - 3.11| = 0.02$cm

$\triangle a_2 = |3.13 - 3.13| = 0$

$\triangle a_3 = |3.13 - 3.14| = 0.01$ cm

$\triangle a_4 = |3.13 - 3.14| = 0.01$ cm

From formula (iii),

$a_{mean} = \frac{0.02 + 0 + 0.01 + 0.01}{4}$ = 0.01 cm

From formula (iii),

% error = $\frac{0.01}{3.13} \times 100$

$= \frac{1}{3.13}$ = 0.3196 .....(using reciprocal table)

= 0.32 %

i. Mean length is **3.13 cm**.
ii. Mean absolute error is **0.01 cm**.
iii. Percentage error is **0.32 %**.

Exercises | Q 3. (vii) | Page 14

Solve the numerical example.

Find the percentage error in kinetic energy of a body having mass 60.0 ± 0.3 g moving with a velocity of 25.0 ± 0.1 cm/s.

SOLUTION

Given: m = 60.0 g, v = 25.0 cm/s, Δm = 0.3 g, Δv = 0.1 cm/s

To find: Percentage error in E

Formula: Percentage error in E

$$= \left(\frac{\triangle m}{m} + 2\frac{\triangle v}{v}\right) \times 100\%$$

Calculation: From formula,

Percentage error in E = $\left(\frac{0.3}{60.0} + 2 \times \frac{0.1}{25.0}\right) \times 100\%$

= 1.3 %

The percentage error in energy is **1.3%**.

Exercises | Q 3. (viii) | Page 15

Solve the numerical example.

In Ohm's experiments, the values of the unknown resistances were found to be 6.12 Ω, 6.09 Ω, 6.22 Ω, 6.15 Ω. Calculate the (mean) absolute error, relative error, and percentage error in these measurements.

SOLUTION

Given: a_1 = 6.12 Ω, a_2 = 6.09 Ω, a_3 = 6.22 Ω, a_4 = 6.15 Ω

To find: i. Absolute error (Δa_{mean})

ii. Relative error

iii. Percentage error

Formulae: 1. $a_{mean} = \frac{a_1 + a_2 + a_3 + a_4}{4}$

2. $\Delta a_n = |a_{mean} - \Delta a|$

3. $\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \Delta a_4}{4}$

4. Relative error = $\frac{\Delta a_{mean}}{a_{mean}}$

5. Percentage error = $\frac{\Delta a_{mean}}{a_{mean}} \times 100\%$

Calculation: From formula (i),

$$a_{mean} = \frac{6.12 + 6.09 + 6.22 + 6.15}{4}$$

$$= \frac{24.58}{4} = 6.145 \text{ cm}$$

Exercises | Q 3. (ix) | Page 15

Solve the numerical example.

An object is falling freely under the gravitational force. Its velocity after travelling a distance h is v. If v depends upon gravitational acceleration g and distance, prove with the dimensional analysis that v = √gh where k is a constant.

SOLUTION

Given = v = $k\sqrt{gh}$

Quantity	Formula	Dimension
Velocity (v)	$\frac{\text{Distance}}{\text{Time}}$	$\left[\frac{L}{T}\right] = [L^1T^{-1}]$
Height (h)	Distance	$[L^1]$
Gravitational acceleration (g)	$\frac{\text{Distance}}{(\text{Time})^2}$	$\left[\frac{L}{T^2}\right] = [L^1T^{-2}]$

k being constant is assumed to be dimensionless.

Dimensions of L.H.S. = [v] = $[L^1T^{-1}]$

Dimension of R.H.S. = $\left[\sqrt{gh}\right]$

$$= [L^1T^{-2}]^{1/2} \times [L^1]^{1/2}$$

$$= [L^1T^{-2}]^{1/2}$$

$$= [L^1T^{-1}]$$

As, [L.H.S.] = [R.H.S.],

$\Rightarrow$ v = k$\sqrt{gh}$ is dimensionally correct equation.

Exercises | Q 3. (x) | Page 15

Solve the numerical example.

$v = at + \frac{b}{t + c} + v_0$ is a dimensionally valid equation. Obtain the dimensional formula for a, b and c where v is velocity, t is time and v_0 is initial velocity.

SOLUTION

Given: $v = at + \frac{b}{t + c} + v_0$

As only dimensionally identical quantities can be added together or subtracted from each other, each term on R.H.S. has dimensions of L.H.S. i.e., dimensions of velocity.

$\therefore$ [L.H.S.] = [v] = $[L^1T^{-1}]$

This means, [at] = [v] = $[L^1T^{-1}]$

Given, t = time has dimension $[T^1]$

$$\therefore [a] = \frac{[L^1T^{-1}]}{[t]} = \frac{[L^1T^{-1}]}{[T^1]} = [L^1T^{-2}] = [L^1M^0T^{-2}]$$

Similarly, [c] = [t] = $[T^1]$ = $[L^0M^0T^1]$

$$\therefore \frac{[b]}{[T^1]} = [v] = [LT^{-1}]$$

$\therefore$ [b] = $[L^1T^{-1}] \times [T^1] = [L^1] = [L^1M^0T^0]$

Exercises | Q 3. (xi) | Page 15

Solve the numerical example.

The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

SOLUTION

Given: l = 4.234 m, b = 1.005 m, t = 2.01 cm = 2.01×10^{-2} m = 0.0201 m

To find: i. Area of sheet to correct significant figures (A)

ii. Volume of sheet to correct significant figures (V)

Formulae: 1. A = 2(lb + bt + tl)

2. V = l × b × t

Calculation: From formula (i),

A = 2(4.234 × 1.005 + 1.005 × 0.0201 + 0.0201 × 4.234)

= 2{[anti log(log 4.234 + log1.005) + antilog(log 1.005 + log0.0201) + antilog(log 0.0201 + log 4.234)]}

= 2{[antilog(0.6267 + 0.0021) + antilog(0.0021 + $\bar{2}.3010$) + antilog ($\bar{2}.3010$ + 0.6267)]}

= 2 {[antilog(0.6288) + antilog ($\bar{2}.3031$) + antilog($\bar{2}.9277$)]}

= 2 [4.254 + 0.02009 + 0.08467]

= 2 [4.35876]

= 8.71752m^2

In correct significant figure,

A = 8.71 m^2

From formula (ii),

V = 4.234 × 1.005 × 0.0201

= antilog [log (4.234) + log (1.005) + log (0.0201)]

= antilog [0.6269 + 0.0021 + $\bar{2}.3032$]

= antilog [0.6288 + $\bar{2}.3032$]

= antilog [$\bar{2}.9320$]

= 8.551×10^{-2}

$= 0.08551 m^3$

In correct significant figure (rounding off),

$V = 0.086\ m^3$

i. Area of a sheet to correct significant figures is $8.72\ m^2$.

ii. ii. Volume of sheet to correct significant figures is $0.086\ m^3$.

Exercises | Q 3. (xii) | Page 15

Solve the numerical example.
If the length of a cylinder is l = (4.00 ± 0.001) cm, radius r = (0.0250 ± 0.001) cm and mass m = (6.25 ± 0.01) g. Calculate the percentage error in the determination of density.

SOLUTION

Given: l = (4.00 ± 0.001) cm,
In order to have same precision, we use,
(4.000 ± 0.001), r = (0.0250 ± 0.001) cm,
In order to have same precision, we use, (0.025 ± 0.001) m = (6.25 ± 0.01) g

To find: percentage error in density

Formulae:

1. Relative error in volume, $\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$ ....(∵ Volume of cylinder, $V = \pi r^2 l$)
2. Relative error $\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$ ....[∵ Density (ρ) = $\frac{\text{mass(m)}}{\text{volume(v)}}$]
3. Percentage error = Relative error × 100 %

Calculation: From formulae (i) and (ii),

$$\therefore \frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

$$= \frac{0.01}{6.25} + \frac{2(0.001)}{0.025} + \frac{0.001}{4.000}$$

= 0.0016 + 0.08 + 0.00025

= 0.08185

% error in density = $\frac{\Delta\rho}{\rho} \times 100$

= 0.08185 × 100

= 8.185%

Percentage error in density is 8.185%.

Exercises | Q 3. (xiii) | Page 15

Solve the numerical example.

When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of Jupiter.

SOLUTION

Given: Angular diameter (α) = 35.72"

= 35.72" × 4.847 × 10^{-6} rad

≈ 1.73 × 10^{-4} rad

Distance from Earth (D) = 824.7 million km

= 824.7 × 106 km

= 824.7 × 109 m.

To find: Diameter of Jupiter (d)

Formula: d = α D

Calculation: From the formula,

d = 1.73 × 10^{-4} × 824.7 × 10^{9}

= 1.428 × 10^{8} m

= 1.428 × 10^{5} km

The diameter of Jupiter is 1.428 × 10^{5} km.

Exercises | Q 3. (xiv) | Page 15

Solve the numerical example.

If the formula for a physical quantity is X = $\frac{a^4b^3}{c^{1/3}d^{1/2}}$ and if the percentage error in the measurements of a, b, c and d are 2%, 3%, 3% and 4% respectively. Calculate percentage error in X.

SOLUTION

Given: X = $\frac{a^4b^3}{c^{1/3}d^{1/2}}$

Percentage error in a, b, c, d is respectively 2%, 3%, 3% and 4%.

Now, Percentage error in X

$$= \left(4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{1}{3}\frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right) \times 100\%$$

$$= \left[(4 \times 2) + (3 \times 3) + \left(\frac{1}{3} \times 3\right) + \left(\frac{1}{2} \times 4\right)\right] \times 100\%$$

= [8 + 9 + 1 + 2] × 100% = 20%

Exercises | Q 3. (xv) | Page 15

Solve the numerical example.

Write down the number of significant figures in the following: 0.003 m^2, 0.1250 gm cm^{-2}, 6.4 x 10^6 m, 1.6 x 10^{-19} C, 9.1 x 10^{-31} kg.

SOLUTION

Number	No. of significant figures	Reason
0.003 m^2	1	Rule no. iii
0.1250 g cm^{-2}	4	Rule no. iv
6.4 × 10^6 m	2	Rule no. i
1.6 × 10^{-19} C	2	Rule no. i
9.1 × 10^{-31} kg	2	Rule no. i

Exercises | Q 3. (xvi) | Page 15

Solve the numerical example.

The diameter of a sphere is 2.14 cm. Calculate the volume of the sphere to the correct number of significant figures.

SOLUTION

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.142 \times \left(\frac{2.14}{2}\right)^3 \quad ...\left(\because r = \frac{d}{2}\right)$$

$$= \frac{4}{3} \times 3.142 \times (1.07)^3$$

$= 1.333 \times 3.142 \times (1.07)^3$

= {antilog [log (1.333) + log(3.142) + 3 log(1.07)]}

= {antilog [0.1249 + 0.4972 + 3 (0.0294)]}

= {antilog [0.1249 + 0.4972 + 3 (0.0294)]}

= {antilog [0.6221 + 0.0882]}

= {antilog [0.7103]}

= $5.133 cm^3$

In multiplication or division, the final result should retain as many significant figures as there are in the original number with the least significant figures.

∴ Volume in correct significant figures = $5.13\ cm^3$